

## 第25回仙台広島整数論集会 アブストラクト

7月7日(火)

10:00 – 10:45 宮崎 雅哉 (九州大学) / Masaya Miyazaki (Kyushu University)

Elliptic curves of rank 2 over the rationals with bad primes containing a prescribed set

In 2025, Zywinina proved that there exist infinitely many elliptic curves of rank 2 over the rationals. It is striking that such a simple fact had not been known until recently. His proof requires the polynomial Szemerédi theorem for primes, proved by Tao and Ziegler in 2008. This theorem is used to keep the number of bad primes of the elliptic curves as small as possible. The elliptic curves he obtained have exactly five bad primes. In this talk, we show that for any given set of odd primes, there exist infinitely many elliptic curves of rank 2 over the rationals whose set of bad primes contains the given set. This result is obtained by applying Zywinina’s method to elliptic curves parametrized by solutions to Pell equations. Since the solutions are determined by the fundamental unit, we can arrange that the set of bad primes contains the given set.

11:00 – 11:45 市川 遼 (東北大学) / Ryo Ichikawa (Tohoku University)

トーションパッケージによる代数曲線の有理点の決定

Determination of rational points on algebraic curves via torsion packets

Understanding the structure of rational points on nonsingular projective algebraic curves over the rational field is a fundamental problem in number theory and arithmetic geometry. Coleman (1985) and Stoll (2006) established, under certain assumptions, upper bounds for the number of rational points, using methods based on  $p$ -adic integration. In this talk, taking a different approach from these methods, we introduce an approach using torsion packets and present our results. As an application, we also discuss the explicit determination of rational points on specific curves.

12:00 – 12:45 大西 達也 (大阪大学) / Tatsuya Ohnishi (The University of Osaka)

一様なパスポートを持つ dessin d’enfant の正則性と自己同型群

Regularity and automorphism groups of dessins d’enfants with uniform passports

For a smooth algebraic curve defined over a number field, one can associate a bipartite graph known as a dessin d’enfant. We investigate the regularity and automorphism groups of dessins d’enfants with uniform passports, that is, those for which the valencies of black vertices, white vertices, and faces are constant, and study how these properties depend on the genus. Although uniformity imposes a high degree of symmetry, such dessins are not necessarily regular. We present an overview of our results and discuss a partial resolution of the conjecture that every uniform passport of genus at least 2 admits a dessin with trivial automorphism group.

14:00 – 14:45 山岸 天馬 (東京理科大学) / Temma Yamagishi (Tokyo University of Science)

虚数乗法をもつ cubic fourfold の超特異還元

On the supersingular reduction of cubic fourfolds with complex multiplication

If  $S$  is a K3 surface with complex multiplication by a CM field  $E$ , Kazuhiro Ito proved that the height of the reduction of  $S$  in characteristic  $p > 0$ , and its Artin invariant if it is supersingular, can be computed from the decomposition of  $p$  in  $E$ . We present analogous results for cubic fourfolds  $X$  with complex multiplication by a CM field  $E$ . We study the rank (which corresponds to Picard number of K3 surfaces), height, and Artin invariant for cubic fourfold in positive characteristic. And we prove that the invariants of mod  $p$  reduction of  $X$  (i.e., whether it is supersingular with its Artin invariant, or non-supersingular with its height) can be computed from the decomposition of  $p$  in  $E$ . Proofs are based on Rikuto Ito’s results on “the main theorem of complex multiplication for cubic fourfold”. We also would like to construct some cubic fourfolds with big Artin invariant.

15:00 – 15:45 武内 竜馬 (東京科学大学) / Ryoma Takeuchi (Institute of Science Tokyo)

A criterion for log regularity via log Frobenius–Witt differentials

Frobenius–Witt differentials, established by T. Saito (2022), play an important role in the theory of characteristic cycles in mixed characteristic. In particular, Saito proved that, for a regular Noetherian scheme  $X$  over  $\mathbb{Z}_{(p)}$  satisfying a suitable finiteness condition, the sheaf  $F\Omega_X$  of Frobenius–Witt differentials is a locally free  $\mathcal{O}_{X_{\mathbb{F}_p}}$ -module of “correct” rank. In this talk, I introduce “log” Frobenius–Witt differentials and give a “log analogue” of the above theorem.

16:00 – 16:45 ミュラー ジョゼフ (国立台湾大学 NCTS) / Joseph Muller (National Taiwan University NCTS)

#### Kraft quivers and twisted Gelfand–Ponomarev modules

Let  $K$  be any field and let  $\sigma, \tau$  be two automorphisms of  $K$ . We consider the  $K$ -algebra  $R := K[F, V]_{\sigma, \tau} / (FV, VF)$  generated by two indeterminates  $F$  and  $V$ , which are respectively  $\sigma$ -linear and  $\tau$ -linear, and which satisfy  $FV = VF = 0$ . Finite dimensional left  $R$ -modules are called “twisted Gelfand–Ponomarev modules”. In this talk, we explain how these can be classified by semilinear representations of Kraft quivers. This is a special case of the classification of modules over semilinear path algebras by “strings” and “bands”, which has been first witnessed by the seminal work of Gelfand and Ponomarev (1968) and Ringel (1975), and later formalized by Butler and Ringel (1987). More recent generalizations are due to Crawley-Boevey (2017), and Bennett-Tennenhaus and Crawley-Boevey (2024) in the semi-linear case. One motivation for classifying modules over  $R$  is the case where  $K$  is perfect of positive characteristic,  $\sigma$  is the Frobenius morphism and  $\tau = \sigma^{-1}$ . In this case,  $R$  is the usual Dieudonné ring modulo  $p$ , and by Dieudonné theory, twisted Gelfand–Ponomarev modules correspond to commutative group schemes over  $K$  which are killed by  $p$ . In 1975, Kraft used this classification to study  $p$ -torsion subgroups of abelian varieties over  $K$ . One other application is to give a direct proof of Theorem 6.1 of Kottwitz and Rapoport’s paper “On the existence of F-crystals” (2003). This talk is based on a survey paper which is joint work with Chia-Fu Yu.

17:00 – 17:45 林 和輝 (東京科学大学) / Kazuki Hayashi (Institute of Science Tokyo)

#### パーフェクトイド塔のエタールコホモロジーの傾化

#### Tilting étale cohomology of perfectoid towers

Since its introduction by P. Scholze, K. S. Kedlaya and R. Liu, the theory of perfectoid spaces has become an effective tool making a bridge between objects in mixed characteristic and those in positive characteristic. One of the most fundamental results in the theory is the étale tilting equivalence, namely, an equivalence between the étale sites of a perfectoid space and its tilt. Although the original proofs relied on analytic methods, K. Česnavičius and Scholze later gave an algebraic proof of the tilting correspondence for the étale cohomology of perfectoid rings. In this talk, we extend this perspective to perfectoid towers connecting Noetherian rings and perfectoid rings. More precisely, we establish a tilting correspondence for the étale cohomology of perfectoid towers.

7月8日 (水)

9:00 – 9:45 根本 裕介 (千葉大学) / Yusuke Nemoto (Chiba University)

#### Transformation formula of Dwork’s $p$ -adic hypergeometric function

Dwork’s  $p$ -adic hypergeometric functions are arithmetically important ( $p$ -adically analytic) functions that describe, for example, the unit roots of elliptic curves of Legendre type. There are so many summation and transformation formulas of hypergeometric functions over the complex numbers and finite fields, however very few such formulas are known for Dwork’s  $p$ -adic hypergeometric functions. Recently, Wang conjectured a transformation formula relating  $t$  and  $\frac{1}{t}$  for Dwork’s  $p$ -adic hypergeometric functions, and proved this conjecture holds in the case where the degree of the hypergeometric function is small. In this talk, we prove that the conjecture holds for general degrees under some assumptions.

10:00 – 10:45 青山 昂平 (大阪大学) / Kohei Aoyama (The University of Osaka)

#### Genus formula for dormant modular curves and towers of function fields

In this talk, we study one-dimensional moduli spaces of dormant  $\mathrm{PGL}_2$ -opers of level  $N$  on pointed stable curves, referred to as dormant modular curves. In the case of 4-pointed genus 0 curves, we give an explicit formula for the genus of the corresponding dormant modular curve. Using this formula, we investigate the asymptotic behavior of the associated towers of function fields and show that they are asymptotically good under suitable conditions. This talk is based on joint work with Yohei Morita and Yasuhiro Wakabayashi.

11:00 – 11:45 椎井 亮太 (九州大学) / Ryota Shii (Kyushu University)

#### A difference formula of $p$ -adic height pairings via the Bloch–Kato logarithm map

Many researchers have introduced and studied  $p$ -adic height pairings, inspired by the formulation of  $p$ -adic analogues of the Birch and Swinnerton-Dyer conjecture. For a Galois representation  $V$  that is semistable at  $p$ ,  $p$ -adic height pairings depend on the choice of splittings of the Hodge filtration on  $D_{\mathrm{dR}}(V)$ . Recently, we obtained a formula for the difference between the  $p$ -adic height pairings of Zarhin–Nekovář arising from two such choices via the Bloch–Kato logarithm map for a 2-dimensional  $p$ -adic Galois representation  $V$  under some assumptions. Under the hypothesis that the localization map  $H_f^1(\mathbb{Q}, V) \rightarrow H_f^1(\mathbb{Q}_p, V)$  at  $p$  is surjective, this formula implies the non-triviality of the  $p$ -adic height pairing. The formula is closely related to a conjecture of Perrin-Riou concerning the relationship between a Heegner point and the Beilinson–Kato element. In this talk, we explain the background of this work and outline the main ideas of the proof. This is joint work with Taiga Adachi (Kyushu University) and Yu Katagiri (Kyushu University).

12:00 – 12:45 進 泰盛 (京都大学) / Taisei Shin (Kyoto University)

Complements of hyperplane arrangements from an anabelian perspective

In anabelian geometry, the case of curves has been extensively studied and anabelian curves are known to be hyperbolic. By contrast, the higher-dimensional case remains far from fully understood. The hyperbolicity of curves is reflected in various conditions arising from topology, algebraic geometry, complex geometry, and group theory. Similar conditions can also be formulated for higher-dimensional varieties. In this talk, I will present results on such conditions for complements of hyperplane arrangements.

14:00 – 14:45 渡部 匠 (東京大学) / Takumi Watanabe (The University of Tokyo)

On an integral structure of the imperfect coefficient ring of  $(\varphi, \Gamma)$ -modules

In the 1980s and 1990s, J.-M. Fontaine established an equivalence of categories between the category of  $p$ -adic Galois representations and the category of étale  $(\varphi, \Gamma)$ -modules. When the  $p$ -adic field is absolutely unramified, following foundational work by Fontaine, N. Wach, and P. Colmez, L. Berger introduced Wach modules, which are special lattices of  $(\varphi, \Gamma)$ -modules, and showed that they characterize  $(\varphi, \Gamma)$ -modules corresponding to crystalline representations. In this talk, we give an explicit description of the intersection of  $A_{\text{inf}}$  with the coefficient ring  $\mathbb{A}_K$  of  $(\varphi, \Gamma)$ -modules, which was remarked by Wach in 1996 without proof. This result indicates the difficulty of generalizing Wach modules to the ramified case.

15:00 – 15:45 梶澤 海斗 (東京大学) / Kaito Masuzawa (The University of Tokyo)

On the correspondence of discrete series representations of  $\text{GSp}(2n)$  and its inner form

The local Jacquet-Langlands correspondence is a bijection between discrete series representations of  $\text{GL}_2(F)$  and those of the multiplicative group of the division algebra over a  $p$ -adic field  $F$ . This correspondence is natural from various aspects. One important property is the character relation. We construct an analogous correspondence for  $\text{GSp}_{2n}(F)$  and its inner form, that is a quaternionic general unitary group, with a character relation. Due to our results, one can also see that Xu's local results partitioning discrete series representations of  $\text{GSp}_{2n}(F)$  into finite subsets is somehow good enough to be said local  $L$ -packets.

16:00 – 16:45 石倉 麟太郎 (東京大学) / Rintaro Ishikura (The University of Tokyo)

On boundedness of the number of cuspidal automorphic representations of  $\text{Mp}(4)$  ordinary at  $p$

The notion of ordinary modular forms, introduced and developed in Hida theory, has played a central role in the study of  $p$ -adic families of automorphic forms. It is therefore natural to ask how this notion extends to automorphic representations of covering groups. This talk is concerned with  $p$ -ordinary cuspidal automorphic representations of the metaplectic group  $\text{Mp}(4)$  with holomorphic discrete series at infinity. Using two Iwahori-level Hecke operators, we define an ordinary projector and investigate the conditions for a representation to be ordinary at  $p$ . Fixing the level  $K$ , we prove a bound (depending only on  $K$ ) for the number of genuine cuspidal automorphic representations of  $\text{Mp}(4)$  ordinary at  $p$  and containing nonzero  $K$ -fixed vectors, as the holomorphic weight varies.

7月9日 (木)

9:00 – 9:45 松村 英樹 (東京都立大学) / Hideki Matsumura (Tokyo Metropolitan University)

Introduction to arithmetic design theory

In algebraic combinatorics, a “design” is a “good” finite set that approximates various symmetric sets (or structures). Motivated by the work of Sawa–Uchida (2019, 2020) on quadrature formulas for the weight functions of classical orthogonal polynomials, research on the interaction between number theory and design theory has recently attracted increasing attention. We refer to this research area as “arithmetic design theory.” In joint work with Masanori Sawa (Ramanujan J., 68(96), 2025), the speaker established a connection between ellipsoidal designs (a generalization of spherical designs on the unit circle) and the Diophantine problem known as the two-dimensional Prouhet–Tarry–Escott (PTE) problem. In this talk, we present joint work with Munenori Inagaki, Sawa, and Yukihiro Uchida that connects combinatorial designs (such as block designs and orthogonal arrays) with higher-dimensional PTE problems (arXiv: 2603.11100).

10:00 – 10:45 柳田 有貴子 (東北大学) / Yukiko Yanagida (Tohoku University)

Estimates for the number of cycles in isogeny graphs

Estimating the number of cycles in a finite graph is important for understanding graph structures and for potential applications to cryptography. In this talk, we focus on supersingular isogeny graphs. To estimate the number of cycles, we use the Ihara zeta function associated with the graphs and the trace formula for Hecke operators.

11:00 – 11:45 井上 博裕 (広島大学) / Hiyu Inoue (Hiroshima University)

Three characterizations of winning strategies in Asymmetric Wythoff's Game: From beatty sequences to continued fractions

We consider a combinatorial game, named Asymmetric Wythoff's Game. Let  $m$  and  $n$  be positive integers. In this game, there are two piles of tokens, and two players move alternately. A player is allowed to take one or more tokens from a single pile, or to take  $i$  tokens from the first pile and  $j$  tokens from the second pile such that  $-m < i - j < n$ . We consider this game under normal play, that is, the player unable to move loses. In this talk, we investigate the positions where the second player has winning strategies (such positions are called  $\mathcal{P}$ -positions). Notice that understanding the set of  $\mathcal{P}$ -position is equivalent to understand a winning strategy: You can win by moving to a  $\mathcal{P}$ -position in your turn. We study  $\mathcal{P}$ -positions from three perspectives. First, we describe the  $\mathcal{P}$ -positions algebraically using a pair of Beatty sequences. Second, we establish a recursive construction using the mex function. Finally, we present an arithmetic characterization based on a Zeckendorf-style numeration system. Notably, we use continued fractions whose period is 2 to resolve the asymmetry of the game, demonstrating a connection between combinatorial game strategies and the theory of continued fractions.

12:00 – 12:45 西淵 裕太 (東北大学) / Yuta Nishibuchi (Tohoku University)

On  $p$ -adic congruences of generalized Euler numbers and relations to even Zeta values

Generalized Euler numbers, defined as the coefficients in the expansion of the reciprocal of an exponential sum, were introduced by Leeming and MacLeod and have been studied for their combinatorial interest. For these numbers, several congruences are known, and in particular, a 3-adic congruence for Lehmer numbers — a special case of generalized Euler numbers — was conjectured by Komatsu. In this talk, we introduce further generalization of these numbers and prove their  $p$ -adic congruences, including the conjecture mentioned above. We also present expressions for the even zeta values in terms of generalized Euler numbers.

14:00 – 14:45 花木 ひかり (東北大学) / Hikari Hanaki (Tohoku University)

The expansion formula for products of Schur multiple zeta functions

The Schur multiple zeta functions introduced by Nakasuji–Phuksuwan–Yamasaki are defined as sums over semi-standard Young tableaux, analogous to classical Schur functions. While the product of Schur functions expands as a linear combination of Schur functions, it is known that a similar expansion for the product of Schur multiple zeta functions can be obtained by taking the summation over all the permutations of the variables. In this talk, we will introduce a refined formula by restricting the summation from the full symmetric group to a specific subgroup.

15:00 – 15:45 中田 怜杜 (名古屋大学) / Reito Nakata (Nagoya University)

多重ゼータ値の  $q$  類似上の導分について

Derivatives of  $q$ -analogues of multiple zeta values

Multiple zeta values are real numbers appearing in various areas of mathematics and theoretical physics. They have various connections to quasi-modular forms, which are holomorphic functions on the complex upper half-plane. To study this connection, one can study  $q$ -analogues, which can be seen as a hybrid object between these two. Quasi-modular forms are known to be closed under taking the derivative, and more precisely they are an example of an  $\mathfrak{sl}_2$ -algebra. In this talk, we discuss a conjecture stating that the space of  $q$ -analogues of multiple zeta values gives a natural extension of this  $\mathfrak{sl}_2$ -algebra.

16:00 – 16:45 范 谷瑜 (名古屋大学) / Ku-Yu Fan (Nagoya University)

Double shuffle relations for integer indices and their application to  $p$ -adic multiple zeta values

This talk first introduces integer indices and explains how the usual admissibility condition for multiple zeta values can be extended to integer indices. We then address the natural question of whether the double shuffle relations are compatible with this extension of admissibility. Answering this question yields double shuffle relations for integer indices. As an application, we extend the definition of  $p$ -adic multiple zeta values to admissible integer indices and show that these extended  $p$ -adic multiple zeta values continue to satisfy the double shuffle relations.

17:00 – 17:45 伊達 聡 (東北大学) / Satoshi Date (Tohoku University)

Anderson–Thakur 級数の Taylor 係数の代数関係式と代数的独立性について

On algebraic relations and independence of Taylor coefficients of Anderson–Thakur series

The Anderson–Thakur series is a lift of the multiple zeta values in positive characteristic defined by Thakur (2004). By considering the prolongation of  $t$ -motives, the Taylor coefficients of the Anderson–Thakur series at  $t = \theta$  can be interpreted as periods of a  $t$ -motive. Papanikolas proved that the algebraic independence of periods of  $t$ -motives is determined by the dimension of the associated  $t$ -motivic Galois group. Matsuzuki (2024) showed the algebraic independence of the Taylor coefficients of the Anderson–Thakur series at  $t = \theta$  for certain cases. In this talk, we discuss the algebraic independence of these Taylor coefficients under relaxed conditions on the indices.

7月10日 (金)

9:00 – 9:45 末吉 翔真 (九州大学) / Shoma Sueyoshi (Kyushu University)

Relation between 2-Selmer groups and ideal class groups associated with Shanks polynomials

Shanks polynomials are cubic polynomials with rational coefficients parameterized by a single rational number. They are known to possess several interesting arithmetic properties, such as their splitting fields generating all cyclic cubic extensions over the rational numbers, and their class numbers being arbitrarily large. In this talk, we focus on the family of elliptic curves defined by these Shanks polynomials and investigate their arithmetic properties. In particular, we will present an explicit equality between the 2-rank of the ideal class group of the splitting field and the dimension of the 2-Selmer group of the associated elliptic curve. Furthermore, using concrete computational examples, we will explain in detail how our work serves as an analogue of the prior results of Chao Li and extends the work of Washington.

10:00 – 10:45 小宮 涼 (東京科学大学) / Ryo Komiya (Institute of Science Tokyo)

On the higher algebraic  $K$ -groups of arithmetically equivalent number fields

We introduce new equivalence relations between number fields called  $K$ -equivalence and almost  $K$ -equivalence, and investigate their relationships with arithmetical equivalence and local integral equivalence. As our main results, by utilizing permutation representations of global Galois groups and the data of local decomposition groups, we refine the previous results by Komatsu in the range  $p \neq 2$  and describe the conditions for number fields to be (almost)  $K$ -equivalent. Through these developments, we clarify how the arithmetic information reflected in higher  $K$ -groups resonates with the special values of zeta functions and Galois representations.

11:00 – 11:45 山田 雄斗 (東京科学大学) / Yuto Yamada (Institute of Science Tokyo)

双対化複体の  $q$  類似

A  $q$ -analogue of dualizing complexes

Garoufalidis–Scholze–Wheeler–Zagier proposed a framework for “arithmetic”  $q$ -analogues based on the Habiro ring. Subsequently, Wagner constructed the Habiro–Hodge complex as a geometric counterpart of this framework. Building on these works, we constructed the Habiro dualizing complex as a  $q$ -analogue of the classical theory of dualizing complexes, which plays a fundamental role in algebraic geometry. In this talk, we give an overview of the background, and then explain the construction of the Habiro dualizing complex together with several basic examples.

12:00 – 12:45 ハマダ ルーカス (東京科学大学) / Lucas Hamada (Institute of Science Tokyo)

Band schemes as relative schemes

Since Y. Manin proposed a strategy to approach the Riemann hypothesis using ideas related to the “field with one element”, several frameworks for this theory have emerged. A central problem is to understand the relationships among these different approaches. The main focus of this talk is on two theories of  $\mathbb{F}_1$  geometry: relative algebraic geometry, namely algebraic geometry over closed symmetric monoidal categories as developed by B. Toën and M. Vaquié, and the theory of bands and band schemes as developed by M. Baker, T. Jin, and O. Lorscheid. After a brief introduction to these theories, I will present a closed symmetric monoidal category, called the category of band modules, whose category of commutative monoid objects is precisely the category of bands. Finally, I will discuss the equivalence between the category of band schemes and the category of schemes relative to the category of band modules.

14:00 – 14:45 寺田 怜央 (九州大学) / Reo Terada (Kyushu University)

Bege Möbius 関数の部分和について

On partial sums of Bege's Möbius functions

In 1970, T. M. Apostol introduced a generalization of the Möbius function. In 2001, A. Bege generalized Apostol's generalized Möbius function. After that, several researchers estimated the partial sums of these functions. In this talk, we present some estimates for the generalized Möbius functions introduced by Apostol and Bege.

15:00 – 15:45 岩田 英人 (群馬工業高専) / Hideto Iwata (National Institute of Technology, Gunma College)

On a relation to the Riemann Hypothesis and an analytic part for the divisor function

Let  $\varphi(n)$  denote the Euler totient function. We study the analytic part associated with the summatory function of  $\sigma_1(n)$  and obtain explicit bounds under the Riemann Hypothesis. In particular, we establish an upper bound of order  $x^{\delta'} \exp\left(\frac{\log x}{\log \log x}\right)$ , where  $\delta' = \max\{1/2, \delta\}$ .