(116) inventam, $a = 1 + \frac{1}{1} + \frac{1}{1.2} + \frac{1}{1.2.3} + \frac{1}{1.2.3.4} + \&c.$, qui termini, si in fractiones decimales convertantur atque actu addantur, præbebunt hunc valorem pro $a =$ $2,71828182845904523536028$, cujus ultima adhuc nota veritati est consentanea. Quod si jam ex hac basi Logarithmi construantur, ii vocari solent Logarithmi *naturales* seu *hyperbolici*, quoniam quadratura hyperbolæ per istiusmodi Logarithmos exprimi potest. Ponamus autem brevitatis gratia pro numero hoc $2,718281828459$ &c. constanter litteram e, quæ ergo denotabit basin Logarithmorum naturalium seu hyperbolicorum, cui respondet valor litteræ $k = 1$; sive hæc littera e quoque exprimet summam hujus Seriei $1 + \frac{1}{1} + \frac{1}{1.2} + \frac{1}{1.2.3} + \frac{1}{1.2.3.4} +$ &c. in infinitum.

123. Logarithmi ergo hyperbolici hanc habebunt proprietatem, ut numeri $1 + \omega$ Logarithmus sit $= \omega$, denotante ω quantitatem infinite parvam, atque cum ex hac proprietate valor $k = 1$ innotescat, omnium numerorum Logarithmi hyperbolici exhiberi poterunt. Erit ergo, posita e pro numero supra invento, perpetuo $e^z = 1 + \frac{z}{1} + \frac{z^2}{1.2} + \frac{z^3}{1.2.3} + \frac{z^4}{1.2.3.4} + \&c.$ ipsi vero Logarithmi hyperbolici ex his Seriebus inveniuntur, quibus est $l(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \frac{x}{6} + \&c$, & $l\,\frac{1+x}{1-x} = \frac{2x}{1} + \frac{2x^3}{3} + \frac{2x^5}{5} + \frac{2x^7}{7} + \frac{2x^9}{9} + \&c.$, quæ Series vehementer convergunt, si pro x statuatur fractio valde parva: ita ex Serie posteriori facili negotio inveniuntur Logarithmi numerorum unitate non multo majorum. Posito namque $x = \frac{1}{5}$, erit $l\,\frac{6}{4} = l\,\frac{3}{2} = \frac{2}{1.5} + \frac{2}{3.5^3} + \frac{2}{5.5^5} + \frac{2}{7.5^7} + \&c.$, & facto $x = \frac{1}{7}$, erit $l\,\frac{4}{3} = \frac{2}{1.7} + \frac{2}{3.7^3} + \frac{2}{5.7^5} + \frac{2}{7.7}$